Evolution Properties of the *y*-Periodic Solitons for the (2+1)-Dimensional Boiti-Leon-Pempinelli System

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With the help of the symbolic computation system Maple and an expanded projective Riccati equation approach, we obtain some new rational explicit solutions with three arbitrary functions for the (2+1)-dimensional Boiti-Leon-Pempinelli system, including Weierstrass function solutions, solitary wave solutions and trigonometric function solutions. From these, several *y*-periodic soliton localized excitations are constructed and some evolution properties of these novel *y*-periodic localized structures are discussed.

Key words: Boiti-Leon-Pempinelli System; Expanded Projective Riccati Equation Approach; Variable Separation Solution; *y*-Periodic Soliton Localized Structure.

1. Introduction

The soliton theory supplies good applications in various fields of natural science, such as in plasmas physics, hydrodynamics, nonlinear optics, fiber optics, solid state physics [1], and the soliton interaction plays an important role in the soliton theory. Therefore, it is of interest to study the interactive properties of solitons. From a symmetry study, we know that there exist more abundant symmetry structures with arbitrary functions for (2+1)-dimensional integrable models [2, 3] than there are in (1+1)-dimensions. Therefore the soliton structure and the interaction between solitons or soliton structures of (2+1)-dimensional nonlinear models may be more complex and show quite rich phenomena that have not yet been revealed. Much work has been done in this respect, and a lot of exciting findings and much progress have been reported recently [4 – 6]. Because the finding of physically relevant soliton solutions in (2+1)-dimensions is much more difficult than that in (1+1)-dimensions, the study of the interaction between solitons in (2+1)-dimensions is very difficult. In this paper, we consider the following celebrated (2+1)-dimensional Boiti-Leon-Pempinelli (BLP) system [7]:

$$u_{ty} = (u^2 - u_x)_{xy} + 2v_{xxx}, \quad v_t = v_{xx} + 2uv_x, \quad (1)$$

which is related to the sine-Gordon equation or the sinh-Gordon equation by certain transformation [7]. Abundant soliton-like, multi-soliton-like and periodic solutions were obtained by using a further extended tanh method [8]. Abundant propagating localized excitations were also derived by Zheng et al. [9] with the help of the Painlevé-Bäcklund transformation and a multi-linear variable separated approach. Here, we discuss its new variable separated excitations with three arbitrary functions via an expanded projective Riccati equation approach, and explore the interaction properties between two *y*-periodic solitons from the derived solutions.

2. New Variable Separated Solutions for the (2+1)-Dimensional BLP System

We apply the expanded projective Riccati equation approach to study (1). We suppose that (1) has formal solutions as follows:

$$\begin{aligned} u(x,y,t) &= \sum_{i=0}^{n} a_i f^i(\xi) + \sum_{i=1}^{n} b_i f^{i-1}(\xi) g(\xi), \\ v(x,y,t) &= \sum_{j=0}^{N} A_j f^j(\xi) + \sum_{j=1}^{N} B_j f^{j-1}(\xi) g(\xi), \end{aligned} \quad (2)$$

where $a_i = a_i(x,y,t)$, $b_i = b_i(x,y,t)$, $A_j = A_j(x,y,t)$, $B_j = B_j(x,y,t)$, with $i = 0, 1, \cdots, n$; $j = 0, 1, \cdots, N$ and $\xi = \xi(x,y,t)$ are arbitrary functions to be determined later. n and N are two positive integers that will be determined soon. $f(\xi)$ and $g(\xi)$ satisfy the projective Riccati equation [10]

$$f'(\xi) = pf(\xi)g(\xi), \quad g'(\xi) = R + pg^2(\xi) - rf(\xi),$$

with

$$g^2(\xi) = -\frac{1}{p}\left(R - 2rf(\xi) + \frac{r^2+\mu}{R}f^2(\xi)\right), \quad (3)$$

where the prime denotes derivative with respect to ξ, $p = \pm 1$, $R\ (\neq 0)$ and r are two constants. Equations (3) admit the following special solutions [11]:

Case 1. If $\mu = -r^2$, (3) has the Weierstrass elliptic function solution

$$f_1(\xi) = \frac{R}{6r} + \frac{2}{pr}\wp(\xi), \quad g_1(\xi) = \frac{12\wp'(\xi)}{R + 12p\wp(\xi)}, \quad (4)$$

where $p = \pm 1$. The Weierstrass elliptic function $\wp(\xi) = \wp(\xi; g_2, g_3)$ satisfies $\wp'^2(\xi) = 4\wp^3(\xi) - g_2\wp(\xi) - g_3$, and $g_2 = \frac{R^2}{12}$, $g_3 = \frac{pR^3}{216}$.

Case 2. If $\mu = h^2 - s^2$ and $pR < 0$, we have the solitary wave solution

$$f_2(\xi) = \frac{R}{r + s\cosh(\sqrt{-pR}\xi) + h\sinh(\sqrt{-pR}\xi)}, \quad g_2(\xi) = -\frac{\sqrt{-pR}}{p}\frac{s\sinh(\sqrt{-pR}\xi) + h\cosh(\sqrt{-pR}\xi)}{r + s\cosh(\sqrt{-pR}\xi) + h\sinh(\sqrt{-pR}\xi)}, \quad (5)$$

where $p = \pm 1$, h and s are arbitrary constants.

Case 3. If $\mu = -h^2 - s^2$ and $pR > 0$, (3) has the trigonometric function solution

$$f_3(\xi) = \frac{R}{r + s\cos(\sqrt{pR}\xi) + h\sin(\sqrt{pR}\xi)}, \quad g_3(\xi) = \frac{\sqrt{pR}}{p}\frac{s\sin(\sqrt{pR}\xi) - h\cos(\sqrt{pR}\xi)}{r + s\cos(\sqrt{pR}\xi) + h\sin(\sqrt{pR}\xi)}, \quad (6)$$

where $p = \pm 1$, h and s are arbitrary constants.

By balancing the highest-order derivative term with the nonlinear terms of (1), we obtain $n = N = 1$. Therefore, ansatz (2) becomes

$$u = a + bf(\xi) + cg(\xi), \quad v = A + Bf(\xi) + Cg(\xi), \quad (7)$$

where a, b, c, A, B, C and ξ are undetermined functions of (x, y, t). $f(\xi)$ and $g(\xi)$ satisfy (3).

With the aid of Maple, substituting (7) along with (3) into (1), collecting all terms with the same power in $f^i(\xi)g^j(\xi)$ $(i = 0,1,2,3,4;\ j = 0,1)$, and setting the coefficients of these terms $f^i(\xi)g^j(\xi)$ $(i = 0,1,2,3,4;\ j = 0,1)$ to zero, we acquire a set of partial differential equations with respect to the unknowns a, b, c, A, B, C and ξ. It is difficult to obtain the general solution of these partial differential equations based on the solutions of (3). Fortunately, in the case of $A = \theta(y)$, $\xi = \chi(x,t) + \psi(y)$, where $\theta \equiv \theta(y)$ is an arbitrary function, $\chi \equiv \chi(x,t)$ and $\psi \equiv \psi(y)$ are two arbitrary variable separated functions of (x,t) and y, respectively, we can get the following results:

Case 1. For $\mu = -r^2$:

$$a = \frac{\xi_t - \xi_{xx}}{2\xi_x}, \quad b = 0, \quad c = -\frac{1}{2}p\xi_x, \quad A = \theta(y),$$
$$B = 0, \quad C = -\frac{1}{2}p\xi_y, \quad \xi = \chi(x,t) + \psi(y). \quad (8)$$

Case 2. For $\mu = h^2 - s^2$:

$$a = \frac{\xi_t - \xi_{xx}}{2\xi_x}, \quad b = 0, \quad c = -p\xi_x, \quad A = \theta(y),$$
$$B = 0, \ C = -p\xi_y, \ r = 0, \ \xi = \chi(x,t) + \psi(y); \quad (9)$$

and

$$a = \frac{\xi_t - \xi_{xx}}{2\xi_x}, \quad b = \pm\frac{1}{2}\sqrt{\frac{p(s^2 - r^2 - h^2)}{R}}\xi_x,$$
$$c = -\frac{1}{2}p\xi_x, \ A = \theta(y), \ B = \pm\frac{1}{2}\sqrt{\frac{p(s^2 - r^2 - h^2)}{R}}\xi_y,$$
$$C = -\frac{1}{2}p\xi_y, \quad r = r, \quad \xi = \chi(x,t) + \psi(y). \quad (10)$$

Case 3. For $\mu = -h^2 - s^2$:

$$a = \frac{\xi_t - \xi_{xx}}{2\xi_x}, \quad b = 0, \quad c = -p\xi_x, \quad A = \theta(y),$$
$$B = 0, \ C = -p\xi_y, \ r = 0, \ \xi = \chi(x,t) + \psi(y); \quad (11)$$

and

$$a=\frac{\xi_t-\xi_{xx}}{2\xi_x},\quad b=\pm\frac{1}{2}\sqrt{\frac{p(s^2-r^2+h^2)}{R}}\xi_x,$$
$$c=-\frac{1}{2}p\xi_x,\ A=\theta(y),\ B=\pm\frac{1}{2}\sqrt{\frac{p(s^2-r^2+h^2)}{R}}\xi_y,$$
$$C=-\frac{1}{2}p\xi_y,\quad r=r,\quad \xi=\chi(x,t)+\psi(y), \tag{12}$$

where $\theta\equiv\theta(y)$ is an arbitrary function, $\chi\equiv\chi(x,t)$ and $\psi\equiv\psi(y)$ are two arbitrary variable separated functions of (x,t) and y, respectively.

From (4)–(6) and (8)–(12), we obtain the following solutions for (1):

Family 1. For $\mu=-r^2$, we obtain the Weierstrass elliptic function solution

$$u_1=\frac{\chi_t-\chi_{xx}}{2\chi_x}-6p\chi_x\frac{\wp'(\xi)}{R+12p\wp(\xi)},\quad v_1=\theta-6p\psi_y\frac{\wp'(\xi)}{R+12p\wp(\xi)}, \tag{13}$$

where $\xi=\chi(x,t)+\psi(y)$, $\chi(x,t)$, $\psi(y)$ and $\theta(y)$ are arbitrary functions of (x,t) and y, respectively, $p=\pm1$, R is an arbitrary constant.

As we know, there is a relation $\wp(\xi;g_2,g_3)=e_2-(e_2-e_3)\mathrm{cn}^2(\sqrt{e_1-e_3}\xi;m)$ between the Weierstrass solution of $\wp'^2(\xi)=4\wp^3(\xi)-g_2\wp(\xi)-g_3$ and the second kind of the Jacobi elliptic function cn, where $m^2=\frac{e_2-e_3}{e_1-e_3}$ is the modulus of Jacobi elliptic functions, e_i ($i=1,2,3$; $e_1\ge e_2\ge e_3$) are roots of $4z^3-g_2z-g_3=0$ [11]. So, the Weierstrass elliptic function solutions u_1 and v_1 can also be written in the form of a Jacobi elliptic function:

$$\begin{aligned}u_{1'}&=\frac{\chi_t-\chi_{xx}}{2\chi_x}-12p\chi_x\frac{(e_2-e_3)\sqrt{e_1-e_3}\mathrm{sn}(\sqrt{e_1-e_3}\xi)\mathrm{cn}(\sqrt{e_1-e_3}\xi)\mathrm{dn}(\sqrt{e_1-e_3}\xi)}{R+12p(e_2-(e_2-e_3)\mathrm{cn}^2(\sqrt{e_1-e_3}\xi))},\\ v_{1'}&=\theta-12p\psi_y\frac{(e_2-e_3)\sqrt{e_1-e_3}\mathrm{sn}(\sqrt{e_1-e_3}\xi)\mathrm{cn}(\sqrt{e_1-e_3}\xi)\mathrm{dn}(\sqrt{e_1-e_3}\xi)}{R+12p(e_2-(e_2-e_3)\mathrm{cn}^2(\sqrt{e_1-e_3}\xi))}.\end{aligned} \tag{14}$$

If $m\to1$, i. e. $e_2\to e_1$, we have $\mathrm{sn}\xi\to\tanh\xi$, $\mathrm{cn}\xi\to\mathrm{sech}\xi$, $\mathrm{dn}\xi\to\mathrm{sech}\xi$, and (14) degenerates into a solitary wave solution:

$$\begin{aligned}u_{1''}&=\frac{\chi_t-\chi_{xx}}{2\chi_x}-12p\chi_x\frac{(e_1-e_3)^{\frac{3}{2}}\tanh(\sqrt{e_1-e_3}\xi)\mathrm{sech}^2(\sqrt{e_1-e_3}\xi)}{R+12p(e_1-(e_1-e_3)\mathrm{sech}^2(\sqrt{e_1-e_3}\xi))},\\ v_{1''}&=\theta-12p\psi_y\frac{(e_1-e_3)^{\frac{3}{2}}\tanh(\sqrt{e_1-e_3}\xi)\mathrm{sech}^2(\sqrt{e_1-e_3}\xi)}{R+12p(e_1-(e_1-e_3)\mathrm{sech}^2(\sqrt{e_1-e_3}\xi))}.\end{aligned} \tag{15}$$

Family 2. For $\mu=h^2-s^2$ and $pR<0$, we have solitary wave solutions:

$$\begin{aligned}u_2&=\frac{\chi_t-\chi_{xx}}{2\chi_x}+p\sqrt{-pR}\chi_x\frac{s\sinh(\sqrt{-pR}\xi)+h\cosh(\sqrt{-pR}\xi)}{s\cosh(\sqrt{-pR}\xi)+h\sinh(\sqrt{-pR}\xi)},\\ v_2&=\theta+p\sqrt{-pR}\psi_y\frac{s\sinh(\sqrt{-pR}p)+h\cosh(\sqrt{-pR}\xi)}{s\cosh(\sqrt{-pR}\xi)+h\sinh(\sqrt{-pR}\xi)},\end{aligned} \tag{16}$$

where $\xi=\chi(x,t)+\psi(y)$, $\chi(x,t)$, $\psi(y)$ and $\theta(y)$ are arbitrary functions of (x,t) and y, respectively, R, h and s are arbitrary constants, $p=\pm1$ and $h^2+s^2\neq0$; and

$$\begin{aligned}u_3=&\frac{\chi_t-\chi_{xx}}{2\chi_x}\pm\frac{1}{2}R\sqrt{\frac{p(s^2-r^2-h^2)}{R}}\frac{\chi_x}{r+s\cosh(\sqrt{-pR}\xi)+h\sinh(\sqrt{-pR}\xi)}\\ &+\frac{1}{2}p\sqrt{-pR}\chi_x\frac{s\sinh(\sqrt{-pR}\xi)+h\cosh(\sqrt{-pR}\xi)}{r+s\cosh(\sqrt{-pR}\xi)+h\sinh(\sqrt{-pR}\xi)},\end{aligned}$$

$$v_3 = \theta \pm \frac{1}{2}R\sqrt{\frac{p(s^2-r^2-h^2)}{R}}\frac{\psi_y}{r+s\cosh(\sqrt{-pR}\xi)+h\sinh(\sqrt{-pR}\xi)} + \frac{1}{2}p\sqrt{-pR}\psi_y\frac{s\sinh(\sqrt{-pR}\xi)+h\cosh(\sqrt{-pR}\xi)}{r+s\cosh(\sqrt{-pR}\xi)+h\sinh(\sqrt{-pR}\xi)}, \tag{17}$$

where $\xi = \chi(x,t)+\psi(y)$, $\chi(x,t)$, $\psi(y)$ and $\theta(y)$ are arbitrary functions of (x,t) and y, respectively, R, h, s and r are arbitrary constants, $p = \pm 1$ and $h^2+r^2 \geq s^2$.

Family 3. For $\mu = -h^2-s^2$ and $pR > 0$, we obtain the following trigonometric function solutions:

$$u_4 = \frac{\chi_t-\chi_{xx}}{2\chi_x} - p\sqrt{pR}\chi_x\frac{s\sin(\sqrt{pR}\xi)-h\cos(\sqrt{pR}\xi)}{s\cos(\sqrt{pR}\xi)+h\sin(\sqrt{pR}\xi)},\quad v_4 = \theta - p\sqrt{pR}\psi_y\frac{s\sin(\sqrt{pR}\xi)-h\cos(\sqrt{pR}\xi)}{s\cos(\sqrt{pR}\xi)+h\sin(\sqrt{pR}\xi)}, \tag{18}$$

where $\xi = \chi(x,t)+\psi(y)$, $\chi(x,t)$, $\psi(y)$ and $\theta(y)$ are arbitrary functions of (x,t) and y, respectively, R, h and s are arbitrary constants, $p = \pm 1$ and $h^2+s^2 \neq 0$; and

$$u_5 = \frac{\chi_t-\chi_{xx}}{2\chi_x} \pm \frac{1}{2}R\sqrt{\frac{p(s^2-r^2+h^2)}{R}}\frac{\chi_x}{r+s\cos(\sqrt{pR}\xi)+h\sin(\sqrt{pR}\xi)} - \frac{1}{2}p\sqrt{pR}\chi_x\frac{s\sin(\sqrt{pR}\xi)-h\cos(\sqrt{pR}\xi)}{r+s\cos(\sqrt{pR}\xi)+h\sin(\sqrt{pR}\xi)}, \tag{19}$$

$$v_5 = \theta \pm \frac{1}{2}R\sqrt{\frac{p(s^2-r^2+h^2)}{R}}\frac{\psi_y}{r+s\cos(\sqrt{pR}\xi)+h\sin(\sqrt{pR}\xi)} - \frac{1}{2}p\sqrt{pR}\psi_y\frac{s\sin(\sqrt{pR}\xi)-h\cos(\sqrt{pR}\xi)}{r+s\cos(\sqrt{pR}\xi)+h\sin(\sqrt{pR}\xi)},$$

where $\xi = \chi(x,t)+\psi(y)$, $\chi(x,t)$, $\psi(y)$ and $\theta(y)$ are arbitrary functions of (x,t) and y, respectively, R, h, s and r are arbitrary constants, $p = \pm 1$ and $h^2+s^2 \geq r^2$.

Because of the arbitrariness of the functions $\chi(x,t)$, $\psi(y)$ and $\theta(y)$ included in the above cases, the physical quantities u and v may possess quite different structures. For example, if $\chi(x,t) = kx+ct$, $\psi(y) = ly$ and $\theta(y) = \lambda y$, all solutions of the above cases become simple traveling wave excitations. Moreover, based on the derived solutions, we may obtain abundant stationary localized solutions, which are not traveling wave excitations or not propagating waves, just as Wu et al. reported about the non-propagating solitons in 1984 [12]. For example, if the arbitrary functions are chosen appropriately, we may derive many kinds of nonpropagating localized solutions like dromion solutions, ring solutions, peakon, compacton solutions [13]. Similarly, several novel typical y-periodic excitations for the physical quantities u and v can also be constructed. We will make a brief discussion about their interaction behavior for the (2+1)-dimensional BLP system in the following.

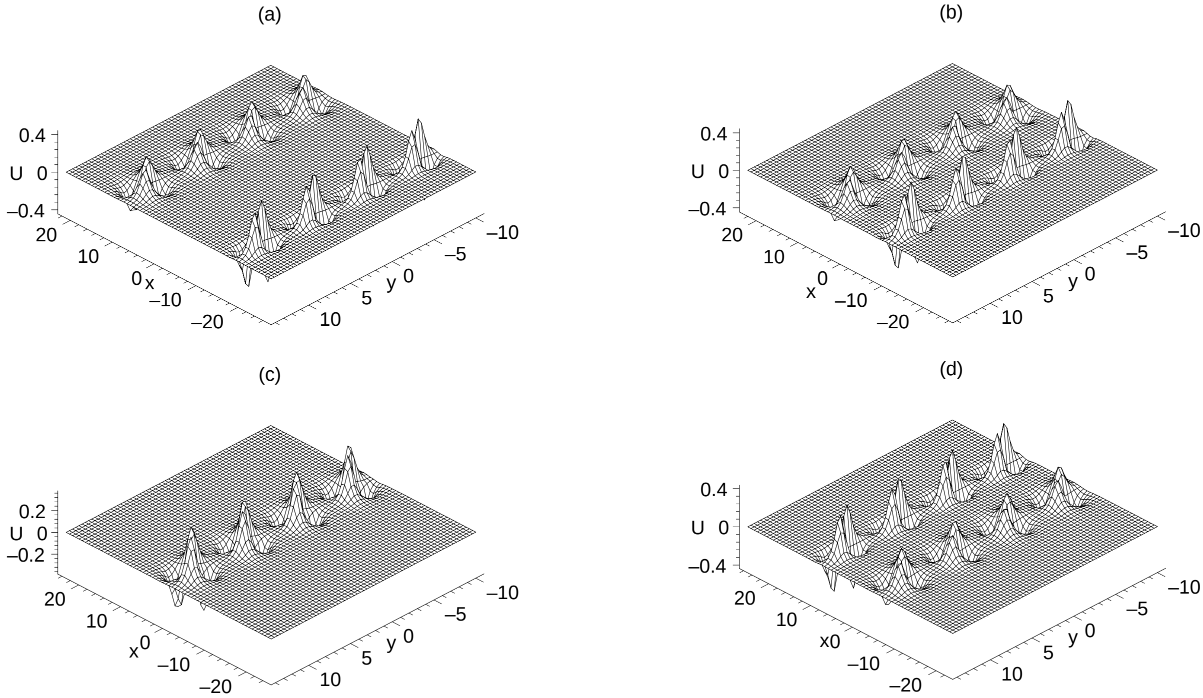
3. Interaction Behavior between Two *y*-Periodic Soliton Localized Structures

In order to construct several kinds of interesting localized y-periodic solitons for the (2+1)-dimensional BLP system, we take the potential function $U \equiv u_y$, where u is expressed by (17). For simplicity, we take $p = -1$, $R = 5$, $h = 1$, $s = 2$, $r = 3$, and one of (17) becomes

$$U \equiv u_y = \frac{5(3+(6-\sqrt{6})\cosh(\sqrt{5}(\chi+\psi))+(3-2\sqrt{6})\sinh(\sqrt{5}(\chi+\psi)))}{2(3+2\cosh(\sqrt{5}(\chi+\psi))+\sinh(\sqrt{5}(\chi+\psi)))^2}\chi_x\psi_y. \tag{20}$$

If the functions $\chi(x+t)$ and $\psi(y)$ are simply chosen as

$$\chi(x+t) = \operatorname{sech}^2(x-t)+\operatorname{sech}(x+0.5t), \quad \psi(y) = \sin y, \tag{21}$$



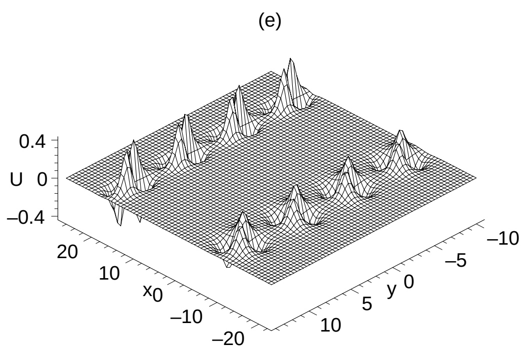


Fig. 1. A plot of the elastic evolution of two rows of y-periodic solitons for the function U expressed by (20) with conditions (21) at different times: (a) $t = -18$; (b) $t = -9$; (c) $t = 0$; (d) $t = 9$; (e) $t = 18$.

we obtain two rows of localized y-periodic solution structures (Fig. 1). From Fig. 1 we conclude that the interaction between the special y-periodic soliton structures is fully elastic (that is, the amplitude, velocity and wave shape of two rows of solitons do not change after their interaction), which is very similar to the completely elastic collision between two classical particles. To show this more carefully, one can find that the position located by the large static localized structure moves from about $x = -18$ to $x = 18$, but its amplitude, velocity and shape are completely preserved after interaction.

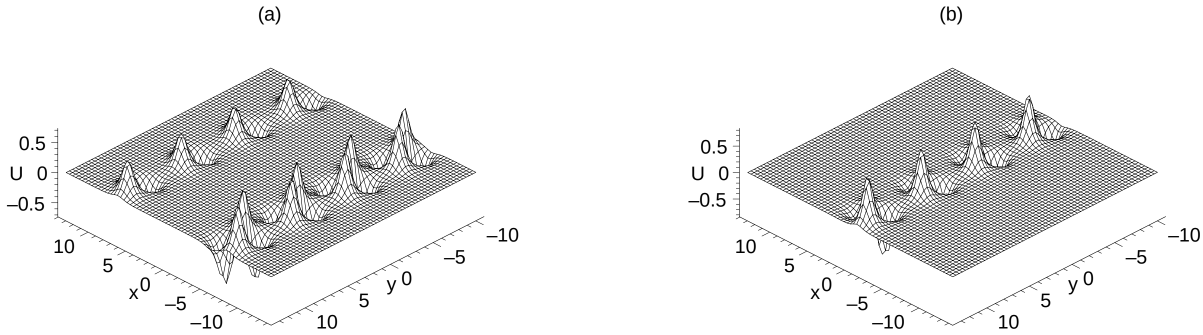
Along with the above idea, however, we find that the interaction between two rows of y-periodic localized structures may exhibit another property, namely that their shapes are not completely preserved after interaction. In order to clarify this nonelastic phenomenon more clearly and visually, two simple examples are depicted in Figs. 2 and 3. Here we take

$$\chi(x+t) = \operatorname{sech}(x-t)^2 + 0.5\tanh(x+t), \quad \psi(y) = \sin y, \tag{22}$$

and

$$\chi(x+t) = \operatorname{sech}(x-t) + 1.8\tanh(x+t), \quad \psi(y) = \sin y. \tag{23}$$

From Fig. 2, we can see that the interaction between two rows of y-periodic localized structures is nonelastic as their amplitudes are changed. In Fig. 3 another



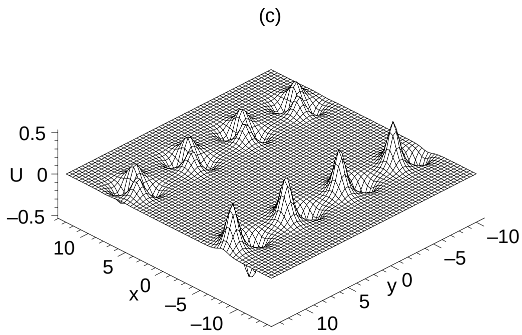
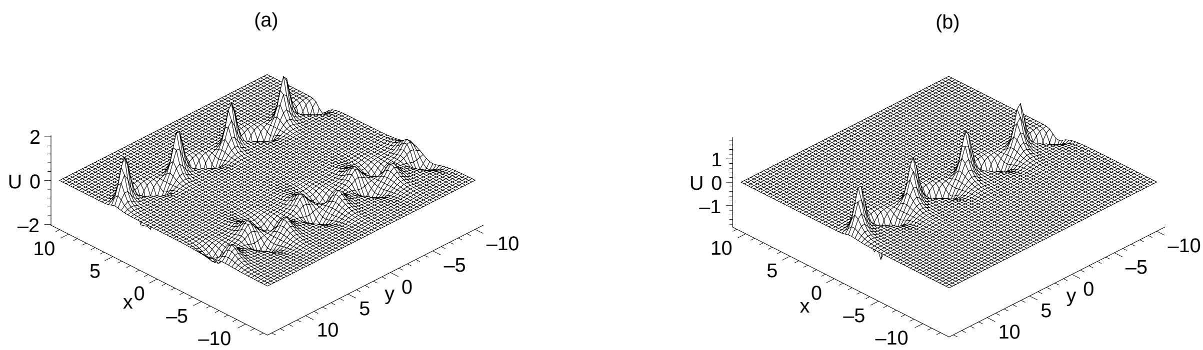


Fig. 2. A plot of the nonelastic evolution of two rows of y-periodic solitons for the function U expressed by (20) with conditions (22) at different times: (a) $t = -7$; (b) $t = 0$; (c) $t = 7$.



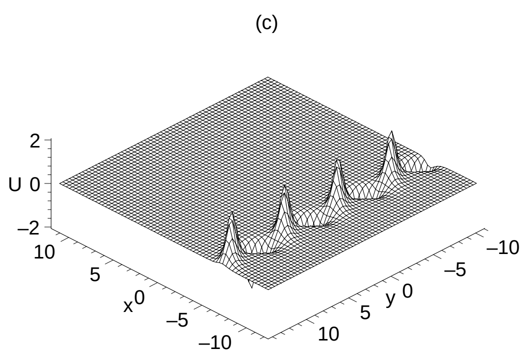


Fig. 3. A plot of the evolution of two rows of y-periodic solitons combined into one for the function U expressed by (20) with conditions (23) at different times: (a) $t = -6$; (b) $t = 0$; (c) $t = 6$.

nonelastic type of y-periodic localized excitations is shown. From Fig. 3, we can see that the two rows of y-periodic solitons are combined into one with increasing time. This phenomenon indicates that their interaction is nonelastic, even a merger of the two rows may happen, which we call completely nonelastic.

4. Summary and Discussion

In summary, with the aid of the symbolic computation system Maple, we obtain some rational explicit solutions of the (2+1)-dimensional BLP system by using the expanded projective Riccati equation method. These solutions include Weierstrass function solutions, solitary wave solutions and trigonometric function solutions. Based on the derived variable separated solutions with three arbitrary functions $[\chi(x+t),\ \psi(y),\ \theta(y)]$, many kinds of localized coherent soliton structures such as multi-dromion, multi-ring, multi-lump solutions, breathers, instantons, peakons, chaotic and fractal soliton solutions can be constructed by choosing the arbitrary functions appropriately. The evolution properties among some y-periodic local structures for the (2+1)-dimensional BLP system are discussed and the results show novel properties and interesting behaviors: the interactions between two rows of y-periodic solitons can be elastic and nonelastic, sometimes even merge. To our knowledge, the interaction properties of the y-periodic localized coherent structures in various types for the (2+1)-dimensional BLP system are first studied and reported here. Thanks to the wide applications of the soliton interactions, to learn more about both other different structures and their interaction behaviors is more relevant. Whether these structures exist or exhibit phenomena in the higher-dimensional nonlinear models are worth studying further.

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